9/12/2015 الأستاذ رياض د. عرفة محاضرة [9]

Realization Structures of digital Filters

(Digital filter structures)

(Structures of IIR Filters)

- Digital filters $\Rightarrow$ discrete time (LTI) systems.
- These filters are described by difference equations.
- These systems can be implemented in hardware or software.
- There are various methods to implement these filters, these methods are called Realization for filter structures.

* IIR filters / FIR Filters
* IIR systems / FIR Systems

IIR $\equiv$ Infinite Impulse response
FIR $\equiv$ Finite Impulse response

IIR Systems/Filters

The general form for the difference equation

$x(n) \rightarrow$ [system/filter] $\rightarrow y(n)$

$$y(n) + a_1 y(n-1) + a_2 y(n-2) + \dots + a_N y(n-N) = b_0 x(n) + b_1 x(n-1) + \dots + b_M x(n-m)$$

FIR systems /Filters

$$y(n) = b_0 x(n) + b_1 x(n-1) + \dots + b_m x(n-M)$$

* for IIR systems / filters

$$Z.T \Rightarrow \left[1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}\right] Y(z)$$

$$= \left[b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m}\right] X(z)$$

$$T.F. = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}}$$

- feedback system
- consists of poles and zeros
- depend on present and past i/ps and past o/ps

*for FIR systems / filters

$$Z.T \Rightarrow T.F. = \frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-M}$$

- The system is not feed back
- consists of zeros only
- depend on the present and past i/ps

Recursive Filters ≡ IIR filters

non-Recursive filters ≡ FIR filters

⟹ Turn over

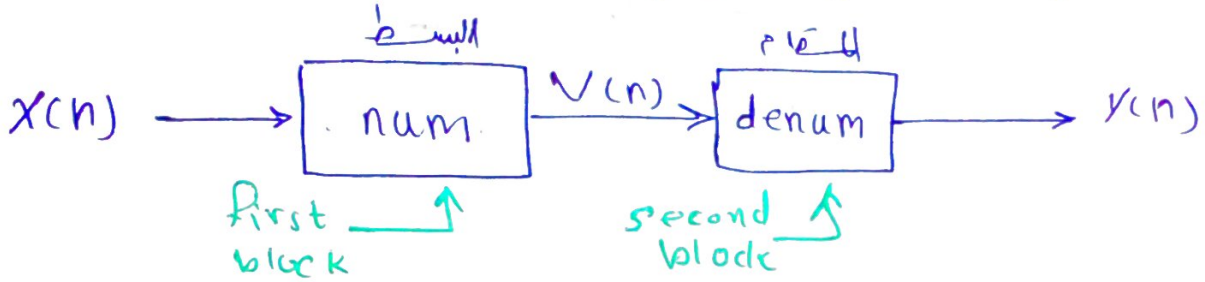
Digital filter structures for IIR Filters (Recursive filters)

1 Direct Form I

$$T.F. = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + b_N z^{-N}}$$

for $N = M = 4$

$$T.F. = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

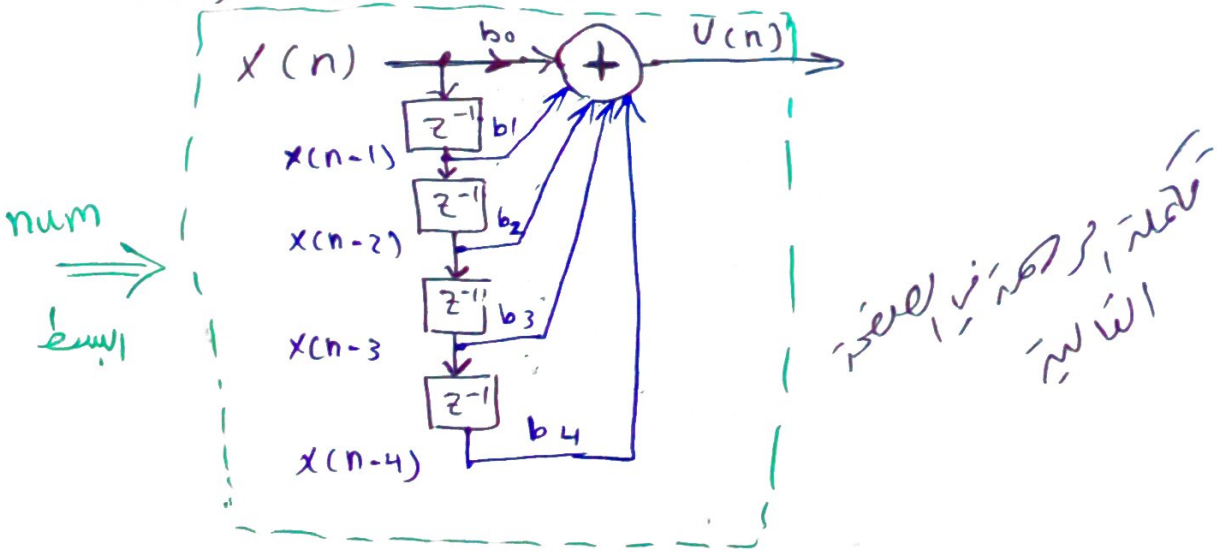


For first block :-

$$\frac{V(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}$$

$$V(z) = \left[b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} \right] X(z)$$

$$V(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + b_3 x(n-3) + b_4 x(n-4)$$



for Second block:

$$\frac{Y(z)}{V(z)} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

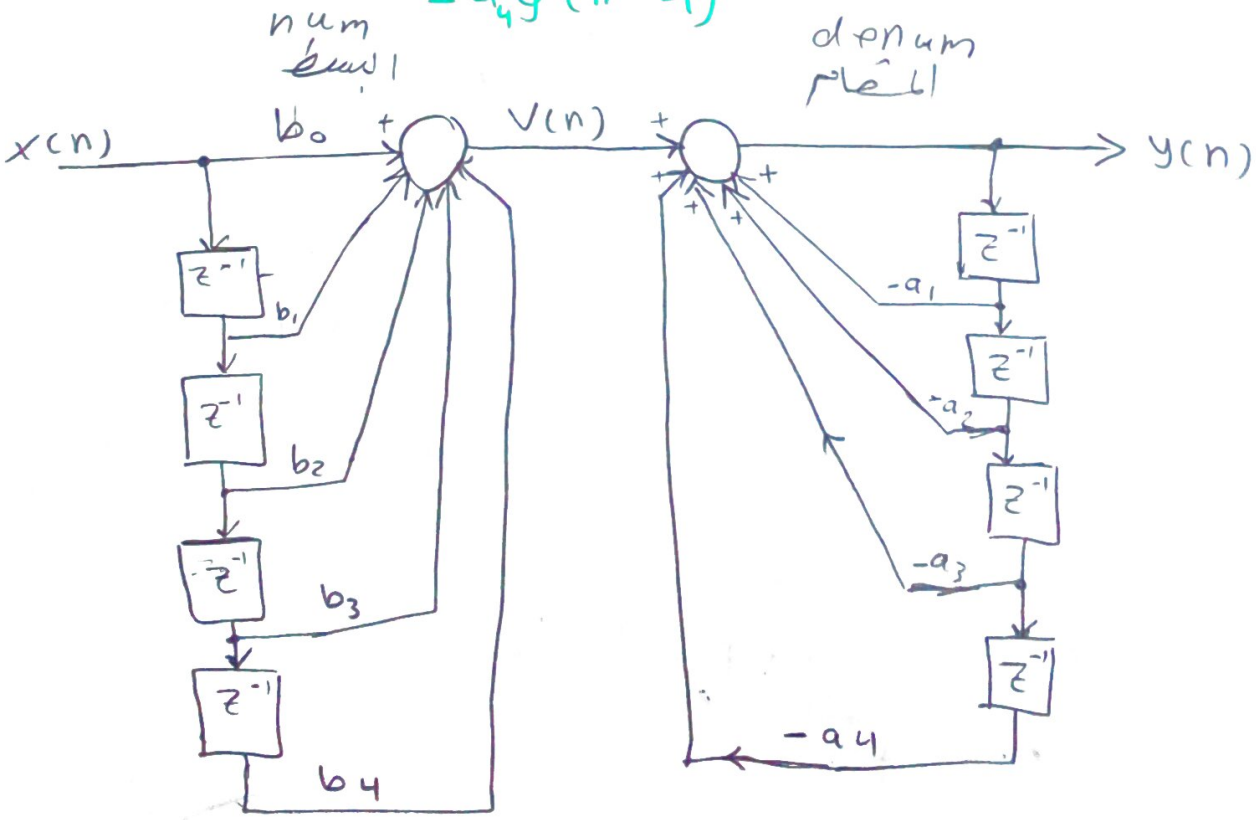
$$V(z) = \left[1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}\right] Y(z)$$

$$Y(z) = V(z) - a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z)$$

$$- a_3 z^{-3} Y(z) - a_4 z^{-4} Y(z)$$

$$y(n) = V(n) - a_1 y(n-1) - a_2 y(n-2) - a_3 y(n-3)$$

$$- a_4 y(n-4)$$



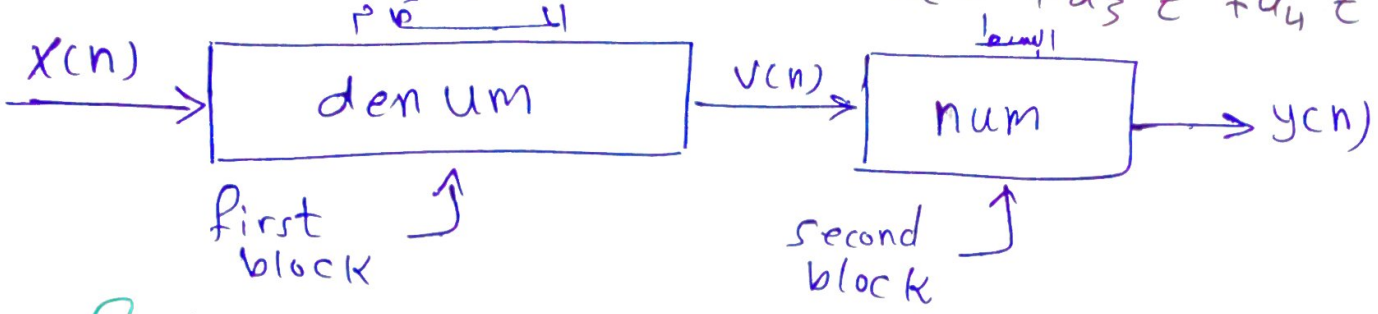
اذا الغير موجود تحذف كوصيلك مع اركم .

⇒ Turn Over

[2] Direct Form II

for ex: $N = M = 4$

$$T.F. = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

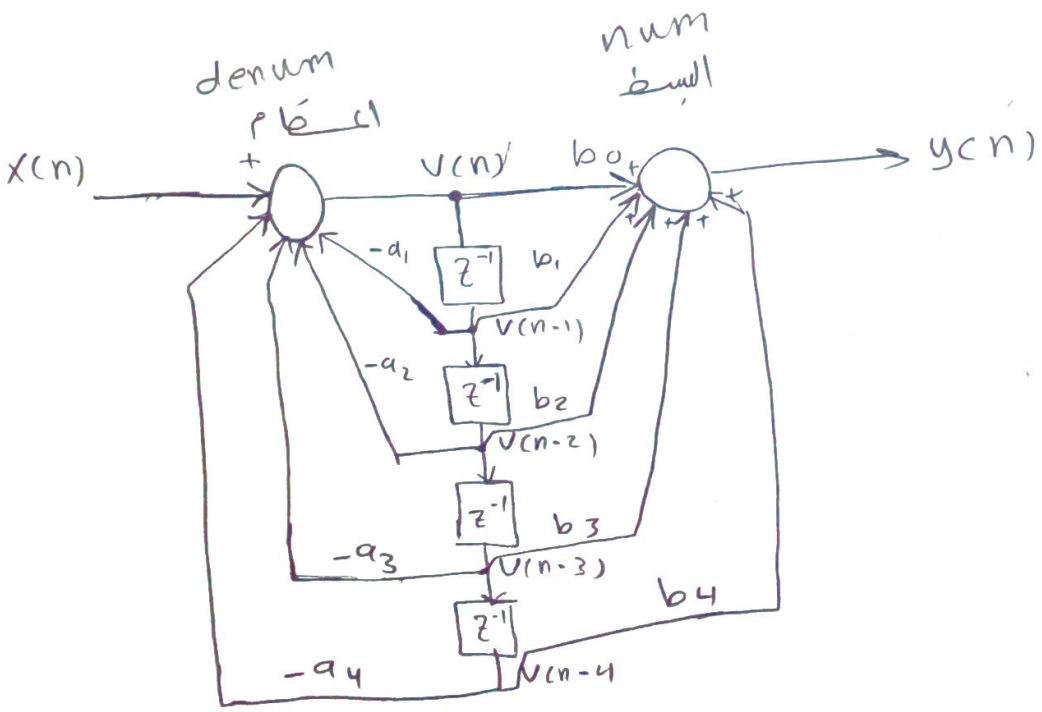


* For first block :-

$$V(n) = X(n) - a_1 (n-1) - a_2 (n-2) - a_3 (n-3) - a_4 (n-4)$$

* For second block :-

$$y(n) = b_0 V(n) + b_1 V(n-1) + b_2 V(n-2) + b_3 V(n-3) + b_4 V(n-4)$$



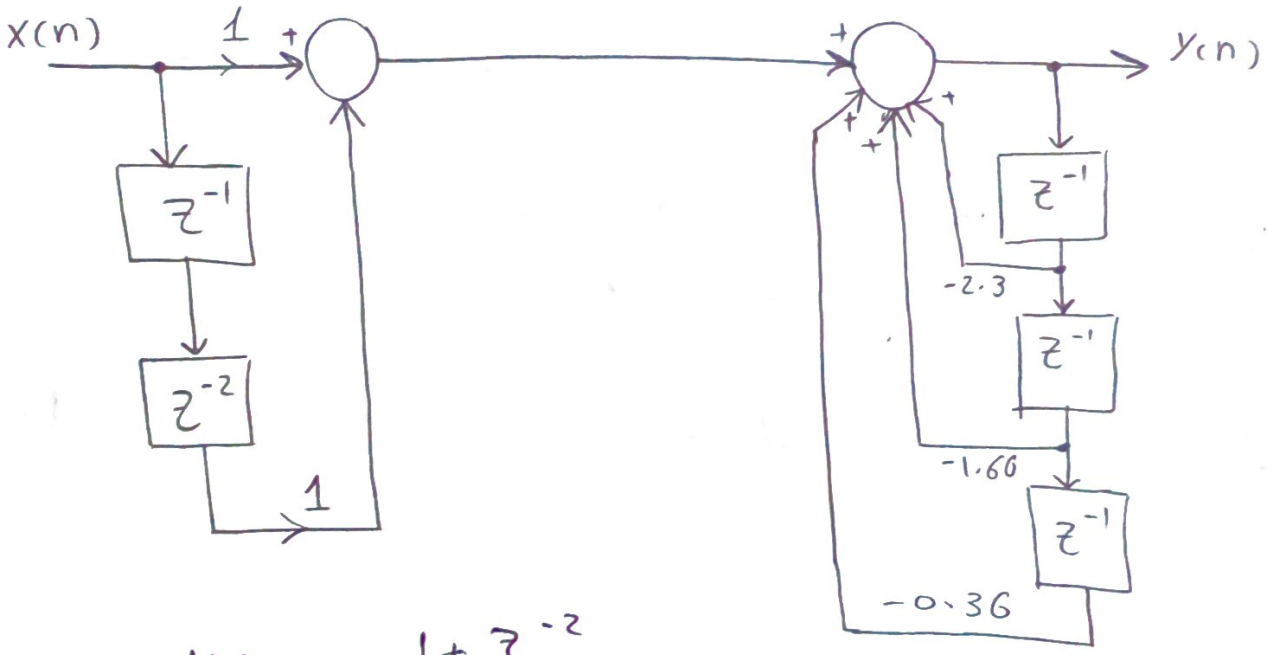
Turn over

Ex: for the following Digital filter

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{(1+z^{-1})(1+0.4z^{-1})(1+0.9z^{-1})}$$

Draw the corresponding Direct form I & II

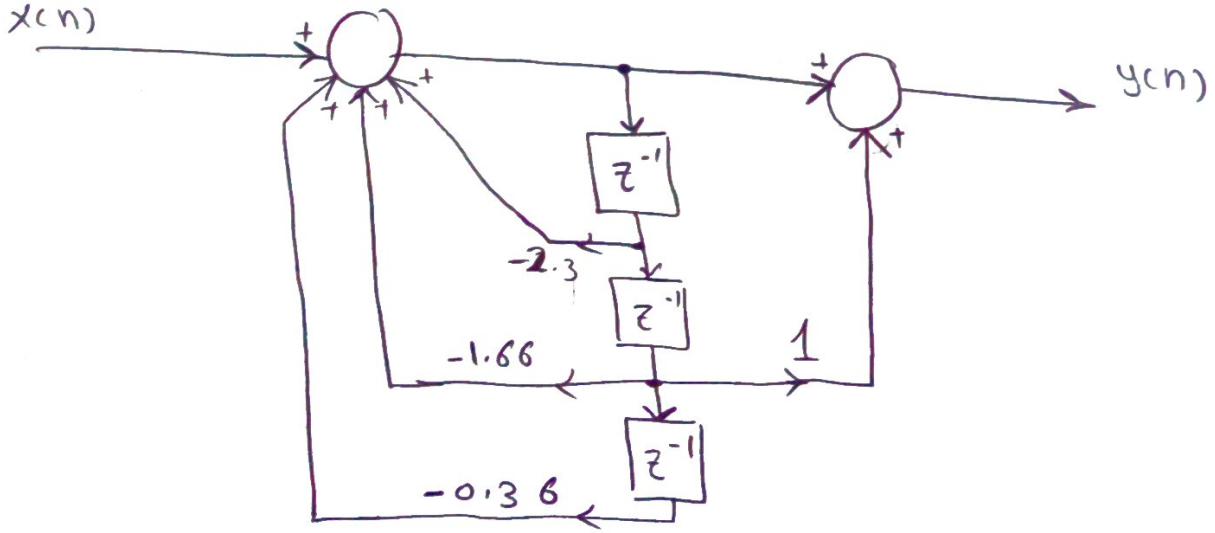
① Direct-form I:



$$H(z) = \frac{1 + z^{-2}}{1 + 2.3z^{-1} + 1.66z^{-2} + 0.36z^{-3}}$$

1 ÷ معاملات المقام

② Direct form II



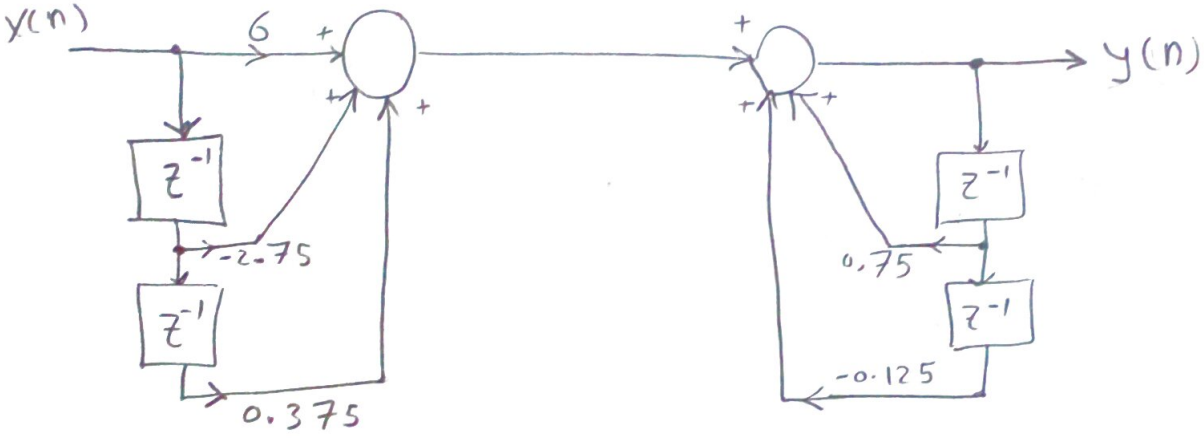
Ex2:

$$H(z) = 3 + \frac{4z}{z-0.5} + \frac{z}{z-0.25}$$

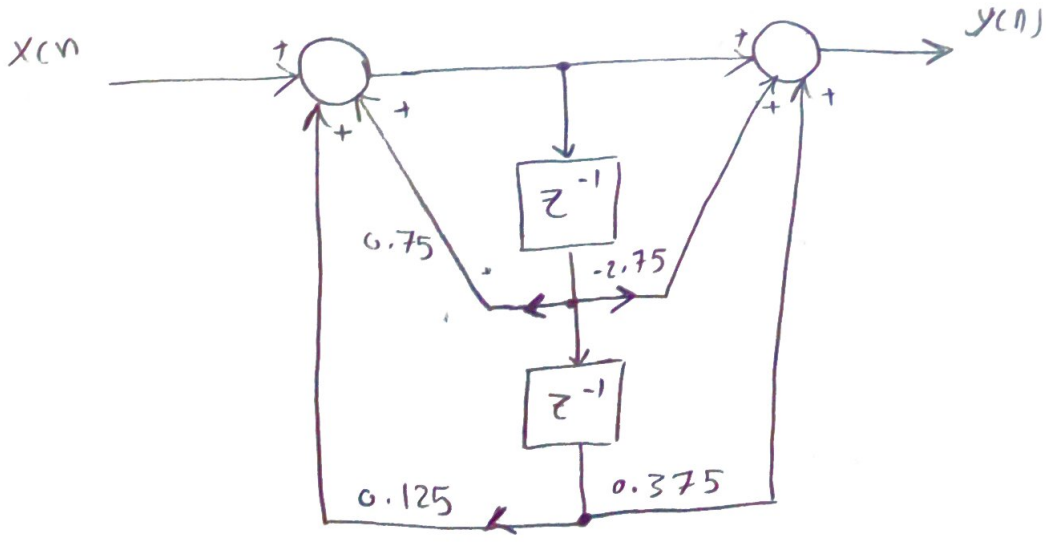
$$= \frac{6z^2 - 2.75z + 0.375}{(z^2 - 0.75z + 0.125)} \times \frac{z^{-2}}{z^{-2}}$$

$$= \frac{6 - 2.75z^{-1} + 0.375z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

Direct Form I:-



Direct form II:-



3] Parallel Form

$$T.F = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots}$$

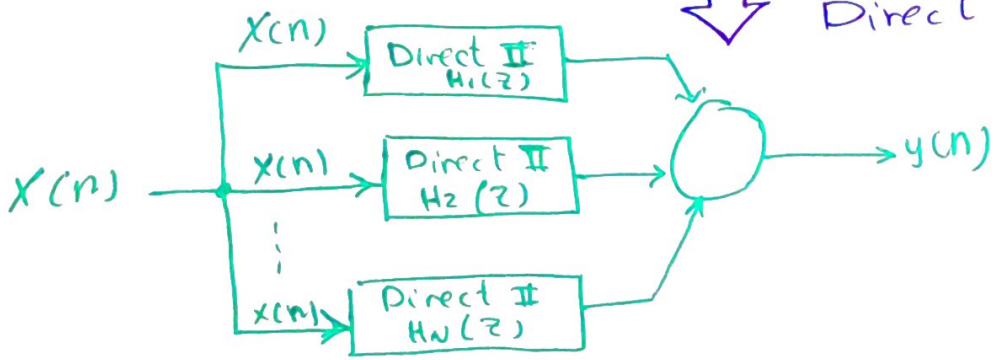
$$= \frac{b_0 + b_1 z^{-1} + \cdots}{(\quad)(\quad)(\quad)\cdots}$$

first or second order ⇓ PF

$$= \frac{A_1}{(\quad)} + \frac{A_2}{(\quad)} + \cdots$$

$$= H_1(z) + H_2(z) + \cdots + H_N(z)$$

Direct form I & II

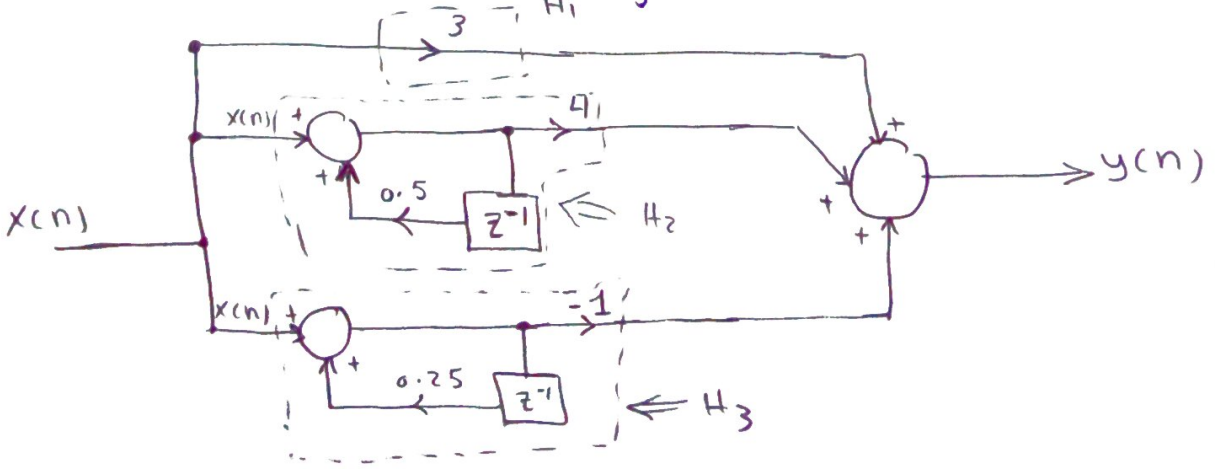


Ex:- $H(z) = 3 + \frac{4z}{z-0.5} - \frac{z}{z-0.25}$

- Realize the System using parallel form

$$H(z) = 3 + \frac{4}{1-0.5z^{-1}} + \frac{-1}{1-0.25z^{-1}}$$

H_1 H_2 H_3



Ex: $H(z) = \dfrac{1 + z^{-2}}{(1 + z^{-1})(1 + 0.4 z^{-1})(1 + 0.9 z^{-1})}$

- Draw the structure form in Parallel form

Put $X = z^{-1}$

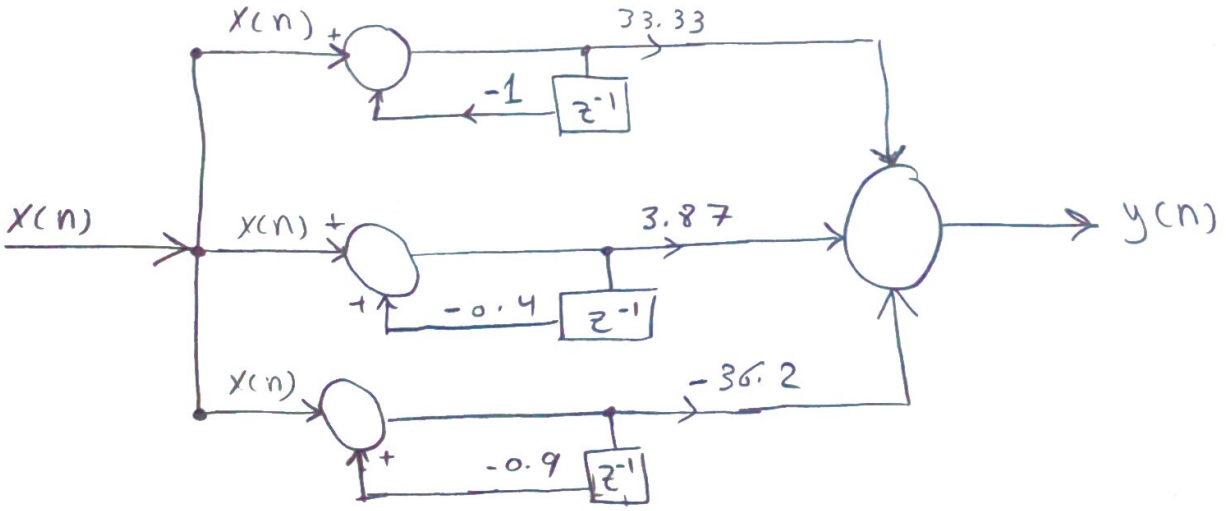
$$H(X) = \frac{1 + X^2}{(1+X)(1 + 0.4X)(1 + 0.9X)} \quad \Downarrow \text{PF}$$

$$= \frac{A_1}{1+X} + \frac{A_2}{1+0.4X} + \frac{A_3}{1+0.9X}$$

$A_1 = \dfrac{1+1}{(1-0.4)(1-0.9)} = \dfrac{2}{0.06} = 33.33$

$A_2 = 3.87$

$A_3 = -36.2$

Put $X = z^{-1}$

$$\therefore H(z) = \frac{33.33}{1 + z^{-1}} + \frac{3.87}{1 + 0.4 z^{-1}} + \frac{-36.2}{1 + 0.9 z^{-1}}$$